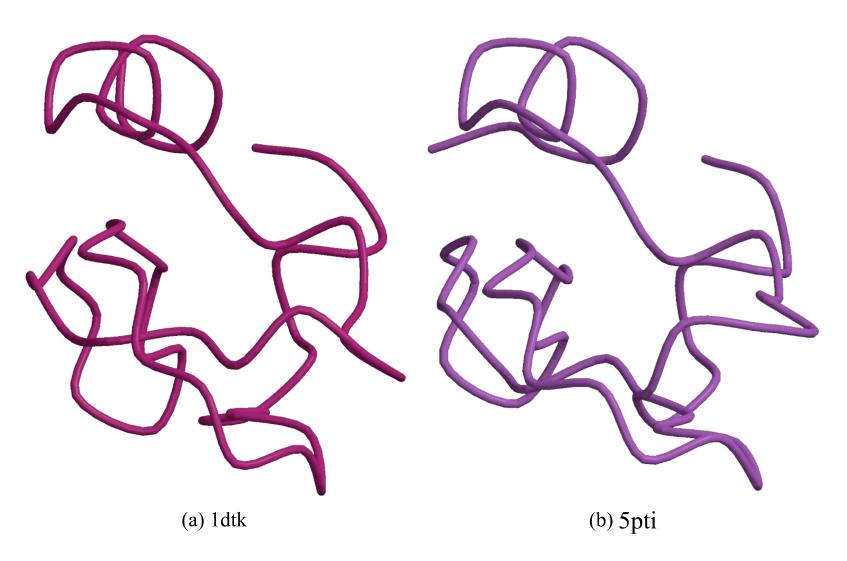
Why compare DNA or protein sequences?

Partial CTCF protein sequence in 8 organisms:

```
H. sapiens
               -EDSSDS-ENAEPDLDDNEDEEEPAVEIEPEPE-----PQPVTPA
P. troglodytes
               -EDSSDS-ENAEPDLDDNEDEEEPAVEIEPEPE-----POPVTPA
C. lupus
               -EDSSDS-ENAEPDLDDNEDEEEPAVEIEPEPE-----PQPVTPA
               -EDSSDS-ENAEPDLDDNEDEEEPAVEIEPEPE-----PQPVTPA
B. taurus
M. musculus
               -EDSSDSEENAEPDLDDNEEEEEPAVEIEPEPE--POPOPPPPPOPVAPA
R. norvegicus
               -EDSSDS-ENAEPDLDDNEEEEEPAVEIEPEPEPOPOPOPOPOPOPAPA
G. gallus
               -EDSSDSEENAEPDLDDNEDEEETAVEIEAEPE-----VSAEAPA
D. rerio
               DDDDDDSDEHGEPDLDDIDEEDEDDL-LDEDOMGLLDOAPPSVPIP-APA
```

- Identify important sequences by finding conserved regions.
- Find genes similar to known genes.
- Understand evolutionary relationships and distances (D. rerio aka zebrafish is farther from humans than G. gallus aka chicken).
- Interface to databases of genetic sequences.
- As a step in genome assembly, and other sequence analysis tasks.
- Provide hints about protein structure and function (next slide).

Sequence can reveal structure



1dtk XAKYCKLPLRIGPCKRKIPSFYYKWKAKQCLPFDYSGCGGNANRFKTIEECRRTCVG-5pti RPDFCLEPPYTGPCKARIIRYFYNAKAGLCQTFVYGGCRAKRNNFKSAEDCMRTCGGA

The Simplest String Comparison Problem

Given: Two strings

$$a = a_1 a_2 a_3 a_4 ... a_m$$

 $b = b_1 b_2 b_3 b_4 ... b_n$

where a_i , b_i are letters from some alphabet like {A,C,G,T}.

Compute how similar the two strings are.

What do we mean by "similar"?

Edit distance between strings a and b = the smallest number of the following operations that are needed to transform a into b:

- mutate (replace) a character
- delete a character
- insert a character

$$riddle \xrightarrow{delete} ridle \xrightarrow{mutate} riple \xrightarrow{insert} triple$$

Representing edits as alignments

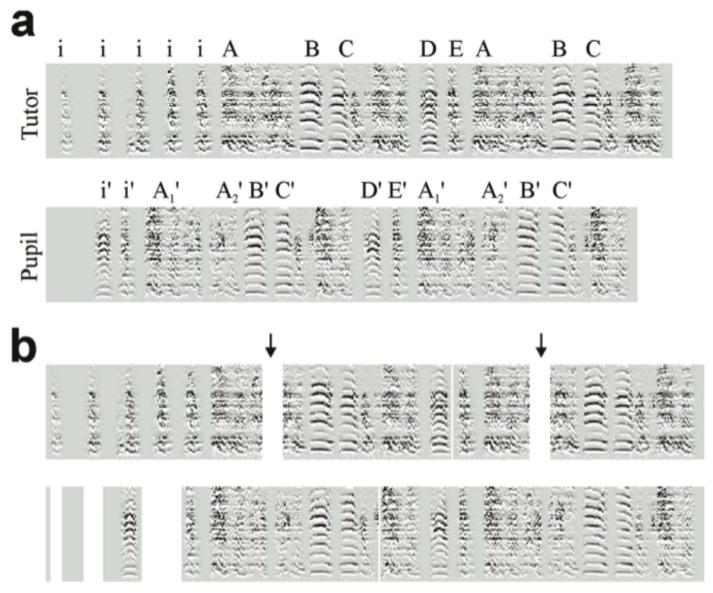
```
prin-ciple
                          prin-cip-le
prinncipal
                          prinncipal-
(1 gap, 2 mm)
                          (3 gaps, 0 mm)
misspell
                          prehistoric
mis-pell
                          ---historic
(1 gap)
                          (3 gaps)
aa-bb-ccaabb
                          al-go-rithm-
                          ababbbc-a-b-
                          alKhwariz-mi
(5 gaps, 1 mm)
                          (4 gaps, 3 mm)
```

NCBI BLAST DNA Alignment

>gb|AC115706.7| Mus musculus chromosome 8, clone RP23-382B3, complete sequence

Query	1650	gtgtgtgtgggtgcacatttgtgtgtgtgtgcgcctgtgtgtg	1709
Sbjct	56838	GTGTGTGGGAAGTGAGTTCATCTGTGTGTGCACATGTGTGCATGCATGCATGTGT	56895
Query	1710	gtg-gggcacatttgtgtgtgtgtgtgtgcctgtgtgtgggtgcacatttgtgtgtg	1768
Sbjct	56896	GTCCGGGCATGCATGTCTGTGTGTGTGTGTGTGTGTGTGTGAGTAC	56947
Query	1769	ctgtgtgtgtgtgcctgtgtgtggggtgcacatttgtgtgtg	1828
Sbjct	56948	CTGTGTGTGTATGCTTGTGTGTGTGTGTGTGTGTGTGTGT	57007
Query	1829	gggtgcacatttgtgtgtgtgtgtgcctgtgtgtgtgggtgcacatttgtgtgtg	1888
Sbjct	57008	TCATCTGTGTGTGTGTGTGTGAGAGTGCATGCATGTGTGTGTGAGT	57055
Query	1889	gcctgtgtgtgtgggtgcacatttgtgtgtgtgtgtgcctgtgtgtgtgggtgcac	1942
Sbjct	57056	TCATCTGTGTCAGTGTATGCTTATGGGTATAACT-TAACTGTGCATGTGTAAGTGTGTTC	57114
Query	1943	atttgtgtgtgtgtgtgtgtgtgtgtgtgtgtgcacatttgtgtgtg	2002
Sbjct	57115	ATCTGTGTATGTGTGTGTGTGAGTTAGTTCATCTGTGTGTGAGAGTGTGTGA	57168
Query	2003	gtgcacatttgtgtgtgtgtgtgtgtgtgtgtgtgtgtgt	2062
Sbjct	57169	GCTCATCTGTGTGAGTTCATCTGTATGAGTGTGTGTATGTGTGTGTACAAATGA	57224
Query	2063	gtgtgtgtgtgtgtgtgtgtgtgtgtgtgtgtgtgtgt	2122
Sbjct	57225	GTTCATCTGTGCATGTGTGTGTTTAAGTGTGTTCATCTGTGTGCGTGT	57274

Comparing Bird Songs



Florian et al. Hidden Markov Models in the Neurosciences

Tracing Textual Influences

Example from Horton, Olsen, Roe, Digital Studies / Le champ numérique, Vol 2, No 1 (2010) She locks her lily fingers one in one. "Fondling," she saith, "since I have hemmed thee here Within the circuit of this ivory pale, I'll be a park, and thou shalt be my deer; Feed where thou wilt, on mountain or in dale:

Graze on my lips; and if those hills be dry, Stray lower, where the pleasant fountains lie." Within this limit is relief enough.... (Shakespeare, Venus and Adonis [1593])

This later play by Markham references Shakespeare's poem.

Common passages identified by sequence alignment algorithms. Pre. Fondling, said he, since I have hem'd thee heere,

VVithin the circuit of this Iuory pale.

Dra. I pray you sir help vs to the speech of your master.

Pre. Ile be a parke, and thou shalt be my Deere: He is very busie in his study. Feed where thou wilt, in mountaine or on dale. Stay a while he will come out anon. Graze on my lips, and when those mounts are drie, Stray lower where the pleasant fountaines lie. Go thy way thou best booke in the world.

Ve. I pray you sir, what booke doe you read? (Markham, The dumbe knight. A historicall comedy... [1608])

The String Alignment Problem

Parameters:

- "gap" is the cost of inserting a "-" character, representing an insertion or deletion
- cost(x,y) is the cost of aligning character x with character y. In the simplest case, cost(x,x) = 0 and cost(x,y) = mismatch penalty.

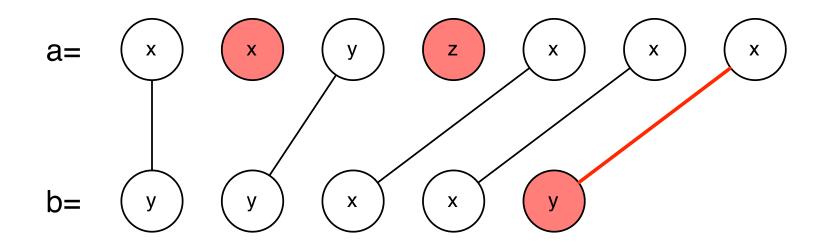
Goal:

- Can compute the edit distance by finding the **lowest cost** alignment.
- Cost of an alignment is: sum of the cost(x,y) for the pairs of characters that are aligned + $gap \times$ number of characters inserted.

Another View: Alignment as a Matching

Each string is a set of nodes, one for each character.

Looking for a low-cost matching (pairing) between the sequences.



Cost of a matching is:

$$gap \times \#unmatched + \sum_{(a_i,b_j)} cost(a_i,b_j)$$

Edges are not allowed to cross!

Algorithm for Computing Edit Distance

Consider the last characters of each string:

$$a = a_1 a_2 a_3 a_4 ... a_m$$

 $b = b_1 b_2 b_3 b_4 ... b_n$

One of these possibilities must hold:

- 1. (a_m, b_n) are matched to each other
- 2. a_m is not matched at all
- 3. b_n is not matched at all
- 4. a_m is matched to some b_j ($j \neq n$) and b_n is matched to some a_k ($k \neq m$).

Algorithm for Computing Edit Distance

Consider the last characters of each string:

$$a = a_1 a_2 a_3 a_4 ... a_m$$

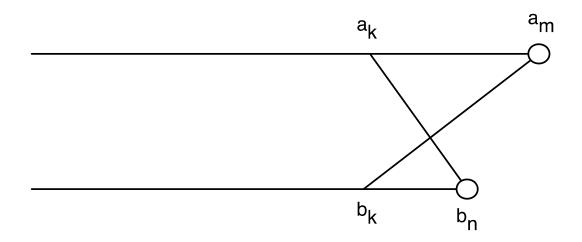
 $b = b_1 b_2 b_3 b_4 ... b_n$

One of these possibilities must hold:

- 1. (a_m, b_n) are matched to each other
- 2. a_m is not matched at all
- 3. b_n is not matched at all
- 4. a_m is matched to some b_j ($j \neq n$) and b_n is matched to some a_k ($k \neq m$).

No Crossing Rule Forbids #4

4. a_m is matched to some b_j ($j \neq n$) and b_n is matched to some a_k ($k \neq m$).



So, the only possibilities for what happens to the last characters are:

- 1. (a_m, b_n) are matched to each other
- 2. a_m is not matched at all
- 3. b_n is not matched at all

Recursive Solution

Turn the 3 possibilities into 3 cases of a recurrence:

$$OPT(i,j) = \min egin{cases} \cos t(a_i,b_j) + OPT(i-1,j-1) & \operatorname{match}\ a_i,b_j \ \operatorname{gap} + OPT(i-1,j) & a_i ext{ is not matched}\ \operatorname{gap} + OPT(i,j-1) & b_j ext{ is not matched} \end{cases}$$
 Cost of the optimal alignment between $a_1...a_i$ and $b_1...b_j$ Written in terms of the costs of smaller problems

Key: we don't know which of the 3 possibilities is the right one, so we try them all.

Base case:
$$OPT(i,0) = i \times \text{gap}$$
 and $OPT(0,j) = j \times \text{gap}$.

(Aligning *i* characters to 0 characters must use *i* gaps.)